possibility of the future, then perhaps Prof. Perry's gloomy picture of the decay of Great Britain may be falsified. The place of our coal-mines in our national assets would then be taken by the vast areas of sunlit land in our Colonial Empire, for fuel-production would then become a question of the number of acre-hours of sunlight available.

I should like to add that what I have said in this letter does not at all lessen the urgency of Prof. Perry's plea for efficient engines; in fact, I think that what I have pointed out tends to strengthen the demand for a great national effort at the sclution of these pressing problems. At present we are, in matters of energy, "robbing posterity" while it is eminently desirable that we should discover a way—if there be one—of "paying our way," and I think that in fuel farming such a way may perhaps be found. WALTER ROSENHAIN.

443 Gillott Road, Edgbaston, April 27.

MR. ROSENHAIN is mistaken as to the ignorance of inventors; many engines have been invented and referred to in newspapers during the last thirty years for utilising solar heat. I may remark that such heat engines may be very efficient, because the available temperature may be very high indeed. I have sometimes wondered why metallurgists neglected the possibility of obtaining very high temperature furnaces from the heat of the sun. As to the energy available, at p. 14 of my book on "Steam" I say:—"On one square foot of Egypt the heat energy received in one year from the sun is about 10^9 foot pounds, or 500 horse-power hours." This is nearly equivalent to the energy of a coating of coal all over Egypt one foot thick, and promises a future for the Sahara and other cloudless regions of the earth. I therefore admit that I did not give sufficient weight to this consideration of the direct heat from the sun, and I am very glad that Mr. Rosenhain has drawn attention to my neglect.

J. PERRY.

Experimental Mathematics.

PROF. PERRY'S syllabus in practical mathematics has now been published two or three years, and the results of actual experience of its working may have some interest. We have in this institute about three hundred students of mathematics, including boys in the day school as well as older evening students, who follow a course on the lines of Prof. Perry's syllabus, and in both classes the adoption of the method has aroused an increased interest in the subject. This increase of interest seems to be due to the fact that the method is essentially experimental as well as deductive. Mathematics is treated as a science rather than, according to a common tradition, as an "arts" subject. The student is taught to investigate the facts for himself by experiment in the form of actual plotting and measurement and numerical calculation, just as in the study of such a science as electricity he investigates a law for himself in the laboratory and, usually at a later stage, proves in his theoretical work that that law follows from his previous knowledge. This is not merely a question of illustrating elementary geometry, but the practice may be carried with advantage into what are usually considered quite advanced parts of his work. However well a student may know the analytical proofs involved, he greatly improves the firmness of his grasp by actually plotting, with various numerical values of the constants, curves to represent such a case, for instance, as the small oscillations of a stiff spring, or the form of a bent beam. In pure mathematics, especially in differential geometry, many examples may be found, and, in fact, the method of conformal representation, which has been so fruitful in the theory of functions and its applications, is really an instance of this method. Besides increasing the average student's interest in his work, these "direct vision" methods, used systematically throughout a student's course, give more solidity and a clearer definition to his ideas than it seems possible to attain by abstract reasoning alone.

My special object, however, in writing is to insist on the value of the method as a logical training. We sometimes hear of the "invaluable logical training" of Euclid with the implied assumption that other methods of treating mathematics are illogical. This view seems to ignore the fact that there is an inductive as well as a deductive logic. If a boy is taught from the beginning to verify all theorems by actual plotting and measurement, he trains, not only his logical powers of deductive reasoning in proving his theorem from its premises, but also his equally logical powers of inductive reasoning from observation and experiment. From the point of view of educational theory this seems a sounder method than to restrict his training to one form of logical reasoning to the neglect of the other. The deductive logic of the syllogism was the only form known in the time of Euclid, but it is scarcely necessary to say that inductive logic now holds a recognised place, and the whole development of modern experimental science has proceeded by its methods.

John Stuart Mill, as is well known, devotes a very scanty consideration to syllogistic reasoning on the ground that "Formal Logic therefore, which . . . have represented as the whole of logic properly so called, is really a very subordinate part of it, not being directly concerned with the process of Reasoning or Inference in the sense in which that process is a part of the Investigation of Truth," and that "The foundation of all sciences, even demonstrative or deductive sciences, is induction."

This may, perhaps, be the explanation of the difficulty which so many boys as well as older students feel in comprehending demonstrative geometry. Most teachers of evening students have met with men of considerable ability and some maturity of mind who have little or no difficulty with algebraical work, but can never comprehend the meaning of a proposition in Euclid. The syllogistic method of reasoning seems to find no avenue into their minds, although they can reason well enough from observed facts. Such people are usually set aside as having no mathematical gift, but all must have notions of space and time, and consequently of change and a rate of change, and if rigid deductive methods were so essential as is often supposed to the science which puts those notions into scientific form, they would scarcely be incomprehensible to so many. If anyone has the power of comprehending the facts of a science such as chemistry, he must have some power of putting that knowledge into scientific form, and so anyone whose experience is given in space and time can scarcely be quite without the power of understanding the science which deals with those conditions of his experience. In fact, if students who seem to be without mathematical power are allowed to approach the subject by an experimental method, they find no difficulty in understanding it and may in time come to grasp the significance of deductive methods. In secondary schools of the classical and mathematical type, boys who are not on the science side are at present almost without the opportunity of developing their inductive logical powers, with the exception of the few who reach the stage where they can draw their own conclusions from the facts of philology or history. Experimental mathematics might in this case be made to supply the place of the missing experimental training.

However one may admire the symmetry of an ideal rigid body of mathematical knowledge, built up in the mind of the learner so that each step is made to depend by flawless abstract reasoning on what has gone before, and so on down to necessary axioms at the foundations, such a process cannot be carried out in the practice of education. It is sometimes said that a student should not be allowed to use any process or to believe any theorem until he can render a complete and perfect reason for it. But if a student is to follow such a method he should not be allowed to use $0\dot{3}$, until he can justify his use of it from a knowledge of the meaning of a limiting value and of the criteria for the convergency of series, nor may he use $\sqrt{2}$ as a number until he has mastered the modern theory of irrational numbers and made up his mind whether to hold opinion with Dedekind and Weierstrass, that the conception of an irrational number is to be based on a purely arithmetical theory, or with Du Bois-Reymond, that it is essentially geometrical and inseparably connected with linear magnitude. It is obvious that no teacher can attempt such a course; these difficulties are always passed over without proof.

In the method of practical mathematics, this practice is frankly recognised as legitimate and natural, and is systematically extended to other parts of mathematics.

Whatever may be true of the superstructure, the fundamental notions of pure mathematics have not been built up by strict deductive process, but by a series of successive approximations to the truth. The conception of a limiting value is a case in point. Until the time of Cauchy, the existence of a limiting value was thought to be self-evident on geometrical grounds in such a case as that of the area of a polygon inscribed in a circle. Cauchy in his treatment of definite integrals recognised that it was necessary to prove that a definite limiting value existed in such a case, but it was only in 1883 that a completely neces-

sary and sufficient criterion for the existence of a definite integral was supplied by Cantor and Dedekind.

Thus the great body of analysis had been built up long before the fundamental notion of a limit was completely established.

A somewhat similar course might be traced in the development of modern ideas as to the basis of mechanics.

In Prof. Perry's method, especially in teaching the calculus, it is recognised that this is the natural way to approach the subject, not only for the science as a whole, but in the mind of the individual student, and its foundations are soundly laid on direct geometrical intuition and the notion of a rate of increase, full analytical treatment being left to a much later stage.

This enables the calculus to be introduced at a much earlier stage than usual, and I may here quote the graphic advice of Prof. Burkhardt, all the more striking as it comes from a mathematician distinguished in pure mathematics:—"Dem angehenden Jünger der Mathematik würde ich raten, sogleich mit beiden Füssen in die Differential- und Integralrechnung zu springen."

F. M. SAXELBY.

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Rearrangement of Euclid's Propositions.

I FEEL that Prof. Lodge's proposal to change the order of Euclid Book I., 1–32, is the real solution of the present problem of the teaching of elementary geometry. The budding engineer has his practical mathematics, the embryo wrangler will absorb geometrical truths served up in almost any manner; but the ninety per cent. to whom mathematics is a mere mental training must have their work put before them in an interesting, practical and yet logical manner. I should, however, like to put forward the following three points:—

(*a*) That Prof. Lodge's idea should be carried further, and Euclid, Books I.–VI., divided into four new books, as:—

The straight line	— Euclid I. 1–32 in some good order.
The circle	— Euclid III. 1–34, IV. 1–5 and escribed circles.
Areas	— Euclid I. 35–48, II., III. 35, 36, 37, IV. 6 to end.
Proportion	— Euclid V. and VI.

For Book I., I would suggest an order commencing with I. 32, cor. 2, which is the most general proposition for all rectilinear figures; and also that certain well-accepted riders should be added, many of which form more powerful instruments for solving geometrical problems than the majority of Euclid's propositions; that, in the circle, tangents should be treated as limiting chords; that, in areas, the "alternative" proofs of Euclid Book II. should be *the* proofs; finally, that proportion be done semi-algebraically, using fractional notation $\frac{AB}{CD}$.

(*b*) That it is not necessary—I may say, not advisable—to teach a beginner the words of a strict definition; but he should be given the idea alone, built up from practical use of a set of instruments, the verbal definition following when he is able to appreciate it. I would advocate that the following definitions be substituted for Euclid's unsatisfactory ones.

A straight line is one such that if any part be taken up and applied to any other part *in any manner*, so that its extremities fall on that part, it will coincide altogether.

The angle—the trigonometrical definition.

Parallel straight lines—the converse of axiom XII.

These would lead, for the student, to the ideas that a straight line can be drawn with a *ruler*, an angle drawn or measured with a *protractor*, and parallel straight lines drawn with two *set-squares*, one fixed and the other movable.

If these were accepted, I. 13, 14, 15, 27, 28, 29 follow almost axiomatically, and we are enabled to prove I. 32, cor. 2, by a supposititious construction, obviating such practical proofs as "walking round the sides" or Prof. Minchin's better idea of placing a pin along a side and moving it round, substituting a purely geometrical proof.

(*c*) That it is unreasonable to bar supposititious proof-constructions—*e.g.* in the bisector proof of I. 5. For no exception is taken to the particular enunciations of I. 4 or I. 8, although at that stage we are unable to *draw* one triangle with its parts equal to those of the other.

J. M. CHILD.

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The Sweet Briar as a Goat Exterminator.

THE introduction of the sweet briar into Australia, in many parts of which it is naturalised, affords a striking illustration of the mode in which the balance of nature may be disturbed in a wholly unforeseen way.

The fruit of the sweetbriar consists of a fleshy receptacle lined with silky hairs which contains the seed-like carpels.

I extract from the *Agricultural Gazette of New South Wales*, vol. xiii., No. 3, March, 1902, p. 313, the following note by Mr. E. A. Weston, a well-known veterinary surgeon of Launceston, Tasmania:—

"With reference to *Rosa rubiginosa*, I thought it might interest you to know that the hairy linings of the fruit caused the death of a number of goats here by forming hairy calculi, which mechanically occluded the lumen of the bowels. These goats were put on the land with the idea that they would eat down the briars and ultimately eradicate them, but the briars came out best and eradicated the goats. The cattle running on the land are also very fond of the briar berries, and from time to time one will die, and on *post-mortem* no pathological changes can be found in any of the organs, nor do the hairy calculi appear in them, although their various stomachs are one mass of the briar seeds."

Kew. W. T. THISELTON-DYER.

Stopping down the Lens of the Human Eye.

IN photography, if the lens is affected with spherical aberration or other defects, or if the aperture is too large for good definition, the operator usually gets over the difficulty by using a smaller aperture or stop. This improves the definition and makes the picture sharp even to the corners of the plate. This process is technically called "stopping down the lens." In amateur landscape work I generally use an aperture or stop with a diameter of one-fiftieth of the focal length of the lens, or *f*/50.

But the human eye has defects, especially as we get old. For instance, the curvature of the crystalline lens becomes too flat, &c., and we have to use spectacles to enable us to read. Reasoning by analogy, diminishing the aperture of the eye by "stopping down the lens" ought to improve defective definition and make the vision sharper, and experiment proves that such is actually the fact. I find that the best effect is obtained by holding a thin metal plate close to the eye, with an aperture in it one-fiftieth of an inch in diameter. This arrangement resembles the old single landscape lens used in photography. The small stop is in front, the lens in the middle and the sensitive plate or retina at the back. I use convex spectacles myself for reading, but with a stop of that size I can easily read small print within 4 inches of the eye (or even less) in a good light without spectacles. I have tried the experiment with several of my elderly friends, and in every case with success. Anyone can try the experiment by means of a pinhole in a card.

I do not know exactly what is the focal length of the lens of the human eye, but supposing it to be half an inch, then with a stop of one-fiftieth of an inch the technical expression for the size of the stop would be *f*/25, or double the diameter of the one I use in landscape photography. I enclose a metal disc with such an aperture. By looking through it I can read the smallest type in NATURE at 4 inches from the eye.

WM. ANDREWS.

Steeple Croft, Coventry, April 25.

Prisms and Plates for Showing Dichromatism.

DICHROMATISM, or the change of colour of an absorbing medium with increasing thickness, is usually shown with plates of coloured glass. It is not always easy to obtain the right kind of glass, and only a few of the aniline dyes are suitable for the purpose. The medium should transmit two distinct regions of the spectrum, the absorption coefficient for one being greater than for the other. I have found that it is better to give the medium the form of a prism, for then the transmitted colours are separated, and the more rapid fading of one as the eye is moved from the refracting edge to the base can be followed. A number of years ago I found a small amount of an unlabeled dye which transmitted a red band and a green band, that is, it had a strong absorption band in the yellow and the blue. Thin layers of this dye were bright green, thick layers were blood red. I have never been able to find the dye again, though I have examined a large number of dyes, but I have found that a mixture of commercial "brilliant green" with a little naphthol yellow has